EFFECT OF DISTRIBUTED INJECTION OF ATOMIC GAS ON THE ELECTRIC AND THERMAL CHARACTERISTICS OF THE POSITIVE COLUMN OF A LOW-PRESSURE DISCHARGE

I. G. Galeev and B. A. Timerkaev UDC 537.525.1

The positive column of a low-pressure electric discharge in the flow of atomic gas in a flat channel with additional injection through the side walls is studied. Expressions for its basic characteristics are derived and analyzed.

The theoretical study of an electric discharge in a gas flow involves the solution of the system of equations describing its electric, thermal, and gas-dynamic characteristics. A procedure has been developed for calculating the electric and thermal characteristics of a longitudinal discharge in the approximation of an incompressible fluid [1-3], which is a rough model when internal heat sources are present. The analysis is often limited to the calculation of separate characteristics of the discharge and their mutual effect is not taken into account [4-6]. In [3] the electric characteristics of the discharge are studied, but the thermal characteristics are not determined. The purpose of this work is to study the behavior of the basic characteristics of a discharge in the flow of an atomic gas and to analyze the effect of different methods of the gas injection on the distribution of the thermal and electric parameters in the discharge zone.

The discharge chamber is a flat channel with porous walls and reticular electrodes placed at the start and at the end of the channel. Gas is pumped in a longitudinal, with respect to the electric field, direction and is additionally injected through the porous walls. The origin of the coordinate systems is chosen to be at the center of the input section of the positive column of the discharge. The x axis is oriented along the discharge chamber D(C), and the y axis is oriented across the discharge chamber.

We write the system of integrodifferential equations describing the positive column of an electric discharge in a flat channel in the following form:

$$D_a N \frac{\partial^2}{\partial y^2}\frac{n}{N} - \frac{g_0}{m}(1+k_w\tilde{x})\frac{\partial}{\partial x}\frac{n}{N} + \frac{g_0 k_w}{m}\tilde{y}\frac{\partial}{\partial y}\frac{n}{N} + \nu_i n = 0, \tag{1}$$

$$I = 2e\mu_e b_z E\int_0^{b_y}\frac{n}{N}\,dy, \tag{2}$$

$$P = NkT, \tag{3}$$

$$c_p g_0(1+k_w\tilde{x})\frac{\partial T}{\partial x} - c_p g_0 k_w \tilde{y}\frac{\partial T}{\partial y} = \lambda\frac{\partial^2 T}{\partial y^2} + k_q e\mu_e E^2\frac{n}{N}. \tag{4}$$

The boundary conditions are:

at the inlet to the discharge zone

$$x = 0 \quad \alpha_e(0,\ y) = \varphi_\alpha(y),\ T(0,\ y) = \theta(y), \tag{5}$$

and at the walls of the DC $y = \pm b_y$, $\alpha_e(x, \pm b_y) = 0$, $T(x, \pm b_y) = \Theta_\omega(x)$.

In writing down the system of equations in the form (1)-(4) we made use of the continuity equation for the mass of the gas

$$\frac{\partial}{\partial x}g_x + \frac{\partial}{\partial y}g_y = 0.$$

Equation (1) is the continuity equation for the electron gas, written taking into account the quasineutrality of the plasma ($n_e \approx n_+ = n$). In writing down (1) the term $\nabla \ln N$ was

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neglected compared with ∇ ln n as was the dependence of the produce $D_\alpha N$ on the temperature of the gas, and the condition $\mu_e >> \mu_+$ was taken into account. In writing down Ohm's law in the integral form (2) it is assumed that the intensity of the electric field depends only on the longitudinal coordinate x. In the conservation equation for the total energy, heat transport by heat conduction along the flow was neglected compared with heat conduction toward the walls and liberation of heat in the volume. For an atomic gas the main fraction of the power injected into the discharge goes into heating the gas $k_q \approx 1$ [6]. The quantities c_p and λ are assumed to be constant here. It is shown in [7] that for a specific volume energy input of $jE \leq 10^6$ W/m³ the computed temperature fields in the region of the discharge in the case when the physical porperties of the gas are temperature dependent are virtually the same as for the case when they are constant. In using the fact that the longitudinal component of the flow density of the mass of the gas $g_x(x)$ from the equations of conservation of the total flow rate and continuity of the gas mass is approximately constant in the transverse cross section of the DC, we find

$$g_x = (1 + k_w\tilde{x})\, g_0,$$
$$g_y = -k_w\tilde{y}\, g_0, \tag{6}$$

where k_w = const is the injection parameter and $g_0 = G/2b_z b_y$ is the flow density of the gas mass in the initial section of the discharge chamber.

The most different approximations of the dependence of the ionization rate on the reduced electric field intensity E/N are used in the literature. Thus, for example, in [3] a quadratic dependence $\nu_i = \varkappa E^2$, where $\varkappa$ is a constant coefficient, is used. In [1] a more general power-law dependence $\nu_i = \varkappa E^m$, where $\varkappa$, m are constants, is used. In addition, in [1, 3] it is assumed that the concentration of neutral particles is approximately constant. Taking into account the fact that N varies, in [8] it is pointed out that the approximation $\nu_i/N = c_\nu^* \cdot (E/N)^m$, where c_ν^*, m are constants, can be used in a quite wide range. In this work the quadratic approximation

$$\frac{\nu_i}{N} = c_\nu \left(\frac{E}{N}\right)^2, \tag{7}$$

where c_ν is a coefficient determined by the type of gas is used.

The solution of the system of equations (1)-(4), satisfying the condition (5), substituting (6) and (7), can be represented in the form

$$E = \left[2\varkappa_E \Phi^2(\tilde{x}) \int_0^{\tilde{x}} \frac{d\tilde{x}}{\Phi^2(\tilde{x})(1 + k_w\tilde{x})} + \frac{\Phi^2(\tilde{x})}{E_0^2\Phi^2(0)}\right]^{-1/2}, \tag{8}$$

$$\alpha_e = \frac{I\sum_m A_m (1 + k_w\tilde{x})^{-\frac{\lambda_m}{k_w \mathrm{Re}_d}} F_m}{2e\mu_e b_y b_z E(\tilde{x})\,\Phi(\tilde{x})}, \tag{9}$$

$$\tilde{T} = \tilde{\theta}_w + \sum_m^\infty F_m^* (1 + k_w\tilde{x})^{-\frac{\lambda_m}{k_w \mathrm{Pe}_d}} \left[\int_0^{\tilde{x}} \frac{H_m(\tilde{x})}{\mathrm{Pe}} (1 + k_w\tilde{x})^{\left(-1+\frac{\lambda_m}{k_w \mathrm{Pe}}\right)} d\tilde{x} + B_m\right]. \tag{10}$$

Here $F_m = F_m\left(\frac{\lambda_m}{2\mathrm{Re}_d k_w}, \frac{1}{2}, -\frac{\mathrm{Re}_d k_w}{2}\tilde{y}^2\right)$ is the confluent hypergeometric function, and

$$\Phi(\tilde{x}) = \sum_{m=1}^\infty A_m (1 + k_w\tilde{x})^{-\frac{\lambda_m}{k_w \mathrm{Re}_d}} \int_0^1 F_m(\tilde{y})\, d\tilde{y},$$

$$A_m = \frac{\int_0^1 \tilde{\varphi}_\alpha(\tilde{y})\,\eta(\tilde{y})\,F_m(\tilde{y})\,d\tilde{y}}{\int_0^1 \eta(\tilde{y})\,F_m^2(\tilde{y})\,d\tilde{y}}, \quad B_m = \frac{\int_0^1 [\theta(\tilde{y}) - \tilde{\Theta}_\omega(0)]\,\eta^*(\tilde{y})\,F_m^*(\tilde{y})\,d\tilde{y}}{\int_0^1 \eta^*(\tilde{y})\,(F_m^*)^2\,d\tilde{y}},$$

$$H_m(\tilde{x}) = \frac{\mathrm{Pe}\int_0^1 \eta^*(\tilde{y})\left[q-(1+k_w\tilde{x})\frac{d\tilde{\Theta}_\omega}{d\tilde{x}}\right]F_m^*\,d\tilde{y}}{\int_0^1 \eta^*(\tilde{y})(F_m^*)^2 d\tilde{y}},\quad q = \frac{b_y k_q e\mu_e E^2}{g_0 T(0,\ 0)c_p}\alpha_e,$$

$$F_m^* = F_m^*\left(\frac{\lambda_m}{2k_w\mathrm{Pe}},\ \frac{1}{2},\ -\frac{\mathrm{Pe}k_w}{2}\tilde{y}^2\right),\ \varkappa_E = \frac{c_v m b_y}{g_0},$$

$$\eta(\tilde{y}) = \exp\left[\frac{k_w\mathrm{Re}_d}{2}\tilde{y}^2\right],\ \eta^*(\tilde{y}) = \exp\left[\frac{k_w\mathrm{Pe}}{2}\tilde{y}^2\right].$$

In the region of the discharge the gas pressure varies insignificantly ($P \approx$ const), so that knowing the distribution of the gas temperature (10) and using (6), it is possible to determine the distribution of the concentrations of the neutral gas particles and the flow velocity along the chamber:

$$N = \frac{P}{kT},\ W_x = \left(1 + k_w\tilde{x}\right)\frac{Tg_0k}{P_m}.$$

The electron density can be found (knowing N), using the expression (9) for the degree of ionization of the gas $\alpha = n/N$. It is convenient to analyze the expressions obtained in particular cases. The expressions (8) and (9) are substantially simplified in the case when the boundary conditions on the degree of ionization of the gas at the inlet into the discharge zone is given in the form $\alpha_e = F\left(\frac{\lambda_1}{2\,\mathrm{Re}_d k_w},\ \frac{1}{2},\ -\frac{k_w\mathrm{Re}_d\tilde{y}^2}{2}\right)$:

$$E = E_{1im}\left[1-\left(1-\left(\frac{E_{1im}}{E_0}\right)^2\right)(1+k_w\tilde{x})^{-\frac{2\lambda_1}{k_w\mathrm{Re}_d}}\right]^{-1/2}, \tag{11}$$

$$\langle \alpha_e \rangle = \langle \alpha_e \rangle_{1im}\frac{E_{1im}}{E}, \tag{12}$$

$$\alpha_{e\,1im} = \frac{E_0}{E_{1im}}\alpha_e(0,\ 0)F\left(\frac{\lambda_1}{2\,\mathrm{Re}_d k_w},\ \frac{1}{2},\ -\frac{\mathrm{Re}_d k_w}{2}\tilde{y}^2\right). \tag{13}$$

Here $E^2{}_{1im} = \lambda_1/\mathrm{Re}_d\varkappa_E$, ⟨ ⟩ denotes an average over the cross section of the DC, and α_{e1im} is the value in the saturation section.

Figures 1 and 2 show the results of computer calculations using the formulas (11) and (13). It is evident from the figures that injection of gas through the side walls of the discharge chamber leads to a more rapid saturation of the electric characteristics along the flow. Injection of gas also increases the nonuniformity of the degree of ionization of the gas in the transverse section. When gas is extracted through the side walls, the profile of the degree of ionization of the gas is smoothed. The change in the relative temperature $(T - \Theta_\omega)/(T(0,\ 0) - \Theta_\omega)$ in the absence of injection ($k_w = 0$) at different distances from the symmetry plane, determined from the expression (10), is shown in Fig. 3. It is evident from the figure that in the region near the wall on the initial section there is a small drop in the temperature, after which the temperature rises and saturates. This explains the predominance of the heat transfer by heat conduction toward the walls on the initial section compared with the liberation of heat in the volume, which leads to cooling of the gas. As the symmetry plane of the DC is approached, the effect of the walls becomes weaker, and the temperature drop in the initial section decreases. In the region near the symmetry plane of the discharge chamber ($y = 0$) the gas temperature rises monotonically owing to the liberation of energy from internal heat sources.

In some cases, in order to obtain a more uniform distribution of the gas dynamic and electric characteristics, the channel is slightly shaped. Let the profile of the chamber be given in the form of the deendence of the half-width of the discharge chamber on the distance along the x axis

$$b_y = \frac{b_y(0)}{f(\tilde{x})},$$

where $f(\tilde{x})$ is some function of $\tilde{x}$.

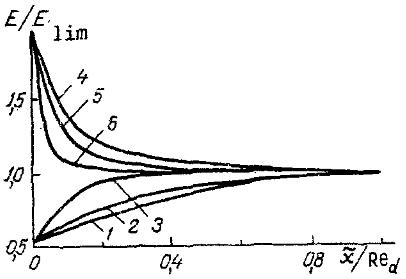

Fig. 1

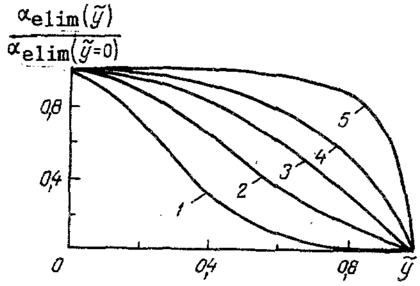

Fig. 2

Fig. 1. Distribution of the electric field intensity along the discharge chamber with distributed gas flow. Curves 1, 2, and 3 were constructed for the values $k_w Re_d$ = 0; 6.42; 19.4, respectively, with E_{lim}/E_0 = 2; curves 4, 5, and 6 were constructed for $k_w Re_d$ = 0; 6.42; 19.4 with E_{lim}/E_0 = 0.5.

Fig. 2. Distribution of the profile of the degree of ionization of the gas in the limiting section of the positive column with gas injection. Curves 1, 2, 3, 4, and 5 were constructed for the values $k_w Re_d$ = 14.5; 4.39; 0; −4.39; −14.5, respectively.

If the boundary condition for the degree of ionization of the gas is represented in the form $\tilde{\varphi}_\alpha = \cos\left(\frac{\pi}{2}\tilde{y}\right)$, the distribution of the electric field intensity and the degree of ionization of the gas in the absence of injection through the wall of the channel can be expressed in terms of their values on the saturation section

$$E = E_s \frac{f(\tilde{x})}{f_s}\left\{1 - \left[1 - \left(\frac{1}{f_s}\frac{E_s}{E_0}\right)^2\right]\exp\left[-\frac{\pi^2}{2\mathrm{Re}_d}\int_0^{\tilde{x}} f(\tilde{x})\,d\tilde{x}\right]\right\}^{-1/2},$$
$$\alpha_e = \frac{E_0}{E} f(\tilde{x})\cos\left[\frac{\pi}{2} f(\tilde{x})\tilde{y}\right]. \qquad (14)$$

Here $E_s = \frac{\pi}{2}\sqrt{\frac{D_a N}{c_v}}\,\frac{f_s}{b_y(0)}$, where f_s is the value far away from the input section. The degree of ionization and the temperature of the gas in the saturation section have the form

$$\alpha_{e_s} = \frac{\langle \alpha_e(0,\ y)\rangle}{E_s/E_0}\,\frac{\pi}{2} f_s \cos\left(\frac{\pi}{2} f_s \tilde{y}\right),$$
$$T_s = \Theta_{\omega_s} + \frac{b_y(0)}{f_s \pi b_z}\,\frac{k_q I E_s}{\lambda}\cos\left(\frac{\pi}{2} f_s \tilde{y}\right).$$

The formulas obtained illustrate the role of shaping of the discharge chamber. Expansion of the discharge chamber along the flow (reduction in the function $f(\tilde{x})$) leads to a drop in the electron density and vice versa. In addition, the distribution of the degree of ionization of the gas becomes more uniform over the cross section of the discharge chamber. The decrease in the width of the channel on the initial section of the DC, as in the case when gas is injected through the wall of the shamber, accelerates the saturation of the electric and thermal characteristics.

Thus the solution of the system of equations (1)-(4), describing the positive column of a discharge in a gas flow, has yielded the distribution of the electric and thermal characteristics of the discharge and their behavior in different states of combustion. The distribution of the electric-field intensity calculated using the formula (14) (with $E_0 = 2.2 \cdot 10^4$ V/m, E_s/E_0 = 0.43, Re_d = 125, f(x) = 1, P = 2 kPa) is compared with the numerical and experimental data, obtained in [9] for a discharge in a hydrogen flow, in Fig. 14. There is good agreement in the character of the distribution of E. This shows that the results of this work will be useful in the design and optimization of electric discharge chambers.

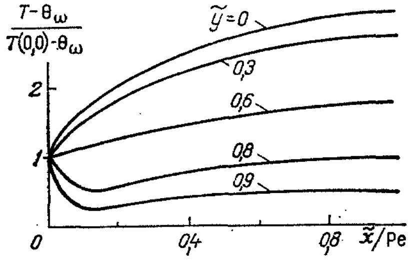

Fig. 3

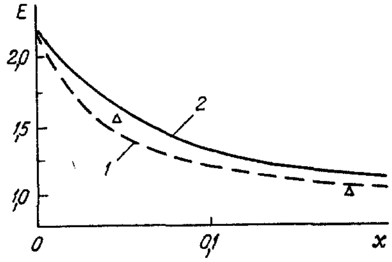

Fig. 4

Fig. 3. Distribution of the relative gas temperature at different distances from the center of the discharge chamber with Po = 10, $E_s/E_0 = 1$, $Pe = Re_d$, θ = const.

Fig. 4. Distribution of the electric field intensity E, 10^4 V/m as a function of the distance x, m, along the discharge chamber: 1) calculation using the formula (14); 2) curve obtained by numerical calculation in [9]; the triangles show the experimental data from [9].

NOTATION

n_e and N, densities of electrons and neutral particles; $\alpha_e = n_e/N$, degree of ionization of the gas; $\langle\alpha_e\rangle$, average value of α_e in the transverse section of the discharge chamber; D_a, coefficient of ambipolar diffusion; μ_e, electron mobility; E, intensity of the electric field; $E_0 = E(0)$; A and $\tilde{A}$, dimensional and dimensionless variables; e, electron charge; ν_i, ionization rate; ρ, P, and T, density, pressure, and temperature of the gas; W, flow velocity vector of the gas; $g_x = \rho W_x$, $g_y = \rho W_y$, components of the mass flux density of the gas; x, y, and z, Cartesian coordinate system; m, particle mass; G, mass flow rate of the gas through the discharge chamber; c_p and λ, heat capacity at constant pressure and the coefficient of thermal conductivity of the gas; k_q, fraction of the power going into heating the gas; I, total electric current; b_z and b_y, width and half-width of the cross section of the discharge chamber along the corresponding coordinate axes; $Re_d = g_0 b_y(0)/D_a N m$, diffusion Reynolds number; $Pe = c_p g_0 b_y(0)/\lambda$, Peclet number; $Po = (b_y)_{lim} k_q I E_{lim}/[b_z \lambda(T(0,\ 0) - \theta_\omega)]$, Pomerantsev number; $\tilde{x} = x/b_y$; $\tilde{y} = y/b_y$; $\tilde{T} = T/T(0,\ 0)$; $\tilde{\alpha}_e = \alpha_e/\alpha_e(0,\ 0)$; and λ_m, m-th root of the degenerate hypergeometric function.

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